Base Film over which Roll Waves Propagate

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The liquid that moves along the duct walls in the annular regime observed in gas-liquid flows often consists of a series of flow surges (roll waves) moving over a thin liquid film (base film). At a given gas velocity there is a critical flow rate of the liquid below which roll waves are not present. Measurements of the height and wall stress are presented to support the notion that the conditions in the base film are close to those which exist at the critical liquid flow rate.

The annular regime observed in gas-liquid or vapor-liquid flow consists of a high velocity of air or vapor in the center of a duct and a thin film of liquid moving along the walls at a much lower velocity than the fluid in the core (*6, 7*). An interchange of droplets between the liquid film and the core often accompanies such flows. Considerable attention has been given to various aspects of this type of flow because of its existence in heat transfer applications, such as the removal of heat from nuclear reactor cores (*17*) and the film cooling of exhaust nozzles (*11, 12*). This paper is concerned with the flow in the liquid on the walls of the duct during annular flow.

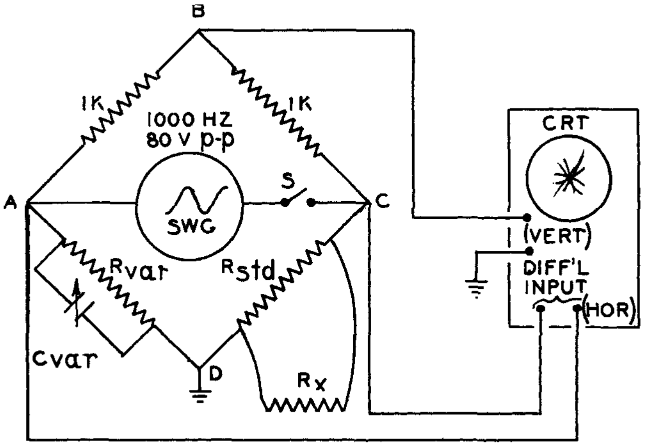
One approach has been to ignore the detailed structure of the film and to assume that the variation of the time averaged velocity is described by relations similar to those which have been used to describe turbulent flow of a single phase close to a wall (*5, 6*). In recent years more information has been obtained about the flow in the liquid film (*7, 8, 16*). It now appears that in many instances the interfacial structure may be characterized by a series of flow surges in the liquid which have a highly agitated surface and which are accompanied by rather sudden increases in the thickness of the liquid layer. These surges have been called *roll waves* by Hanratty and Hershman (*8*) in their investigation of the stability of relatively thick films, and *disturbance waves* by Taylor (*16*) in his studies of air-water flow in a vertical pipe.

These observations suggest that it might be advantageous to consider a more detailed model of the film that separately accounts for the roll waves and the slower fluid layer over which the roll waves appear to be moving. This paper describes the results of a study of the base film in a horizontal enclosed rectangular channel. Water was flowed on the bottom wall and air was blown over its surface at large enough velocities to generate roll waves. Techniques have been developed to measure the height, h_o, and the velocity gradient at the wall for the base film separately from the roll waves. From these measurements $h_o^+ = h_o u^*/\nu$ can be calculated. For very thin films a linear velocity profile can be assumed and therefore the mass flow can be calculated.

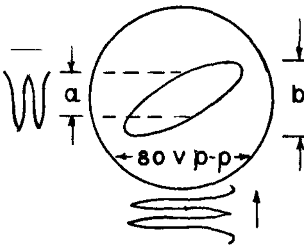
EXPERIMENTAL PROCEDURE

The experiments were conducted in a channel which is 12 in. wide, 0.982 in. high, and 18.5 ft. long. It is preceded by an 11 ft. section which gradually converges from a height of 6 in. to the height of the test section in order to minimize entrance effects associated with the manner in which the phases are initially contacted. Distilled water is introduced at the beginning of the entry section through a series of screens on the bottom of the channel. Filtered air is blown over the top of the water. Measurements were made at a location 14.5 ft. from the beginning of the test section.

The height of the base film between the roll waves was determined by measuring the conductivity of the liquid film. This technique had been used earlier by a number of experimenters (*9, 10, 15*). In this particular application the conductivity cell consists of two circular stainless steel electrodes in the wall beneath the liquid. The electrodes are 1 in. apart and they are oriented so that their line of centers are perpendicular to the direction of flow. The resulting cell is placed in parallel with one arm of a manually balanced a.c. Wheatstone bridge. Since the electrodes are aligned parallel to the roll waves and since the roll waves are often relatively far apart, it is possible to distinguish between the bridge null fluctuations caused by the roll waves and caused by the base film.



(a) BRIDGE AND DETECTOR



(b) OSCILLOSCOPE DISPLAY

Fig. 1. Conductivity bridge and null detection arrangement.

This is done by displaying a Lissajous figure of the bridge imbalance signal on a cathode ray oscilloscope. Figure 1 shows a sketch of the bridge and null detection circuit and of the Lissajous trace. The bridge imbalances a' and b' are known functions of the phase angle (*14*). The capacitive imbalance is nulled by adjusting C_{var} until $a' = 0$. The magnitude of b' is then a measure of the resistive imbalance. After balancing the capacitance of the unknown, a flattened ellipse is observed to see-saw back and forth about the center of the oscilloscope screen. Since the liquid film consists largely of the base film the ellipse see-saws through a small angle most of the time. The resistance, R_{var}, is adjusted so that the horizontal approximately bisects this small angle. When a roll wave passes, the ellipse rotates a distance away from the horizontal and stays there for the small period during which the roll wave is passing. The brightest traces usually correspond to the base film since they repeat more times than those imbalances due to roll waves. The ratio of the resistance

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of the channel solution in a standard cell to the measured operating resistance, $R_x = R_{std} R_{var}/R_{std} - R_{var}$, can be directly related to the film height from a static calibration.

The velocity gradient at the wall was measured using a technique proposed by Ludwieg (*13*) and recently used by Bellhouse and Schultz (*1*). The electrical power supplied to a small heated element mounted flush in a duct wall varies linearly with the third root of the velocity gradient at the wall if certain constraints are maintained in probe design and operation. A constant temperature difference must be maintained between the temperature of the heated element, T_f, and the fluid temperature upstream of the probe, T_L. The heated element must be thin enough that the thermal boundary layer is contained within a region where the velocity profile is described by a linear relation. Bellhouse and Schultz (*1*) have shown that the expression

$$\tau_w^{1/3} = C_1 \frac{I^2 R_f}{(T_f - T_L)} + C_2 \qquad (1)$$

describes the response in air when $(T_f - T_L)$ is varied less than 4°C. and the constants C_1 and C_2 are obtained from calibration.

In this study a DISA 55A01 constant temperature anemometer maintained a DISA 55A93 subminiature flush-mounted hot-film probe at a constant temperature. The hot-film probe was calibrated directly in the apparatus by measuring the current to the heated element when a liquid film flowed laminarly under the traction of a turbulent air stream. By using a 0.01% solution of sodium lauryl sulfate in distilled water the initiation of waves on the horizontal film was retarded and it was possible to have a laminar film at relatively high gas velocities. The shear stress at the wall was calculated from the measured pressure drop in the channel. The calibration curve was found to agree with Equation (1). Relatively low overheats had to be used in order to avoid problems associated with the evolution of bubbles at the surface of the probe.

RESULTS

Measured base film heights, h_o, are presented as a function of the superficial gas velocity in Figure 2. The parameter w represents the total mass flow rate in both the roll waves and the base film. The interesting feature is the relative insensitivity of the base film height to increases in the liquid throughput. This would seem to imply that flow in the base film remains constant and that the additional liquid is accommodated by an increase in the amount of liquid transported by roll waves.

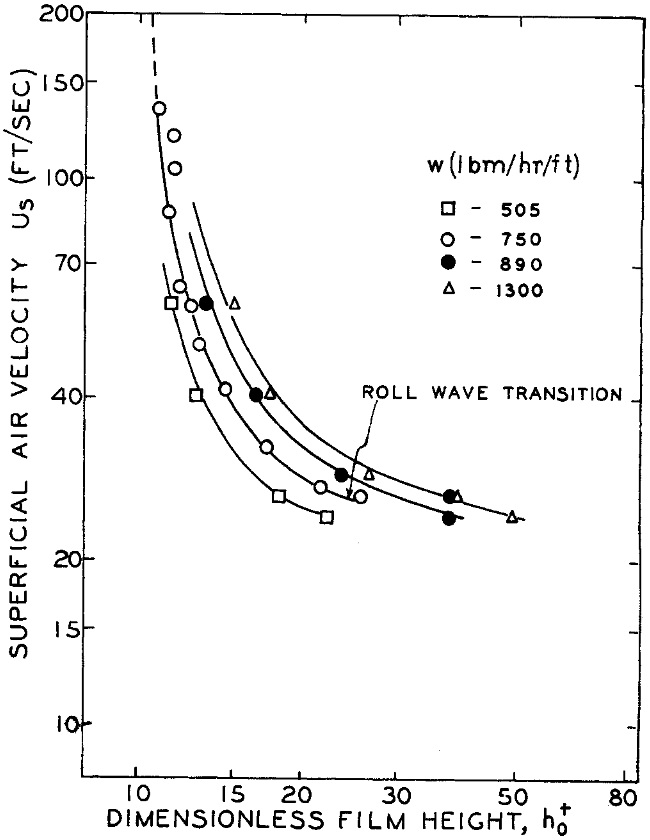

Fig. 3. Dimensionless base film heights.

The roll waves moving over the base film are much higher than the base film. In order to illustrate this difference the approximate average heights of the roll waves reported by one of the authors (*18*), are sketched in Figure 2.

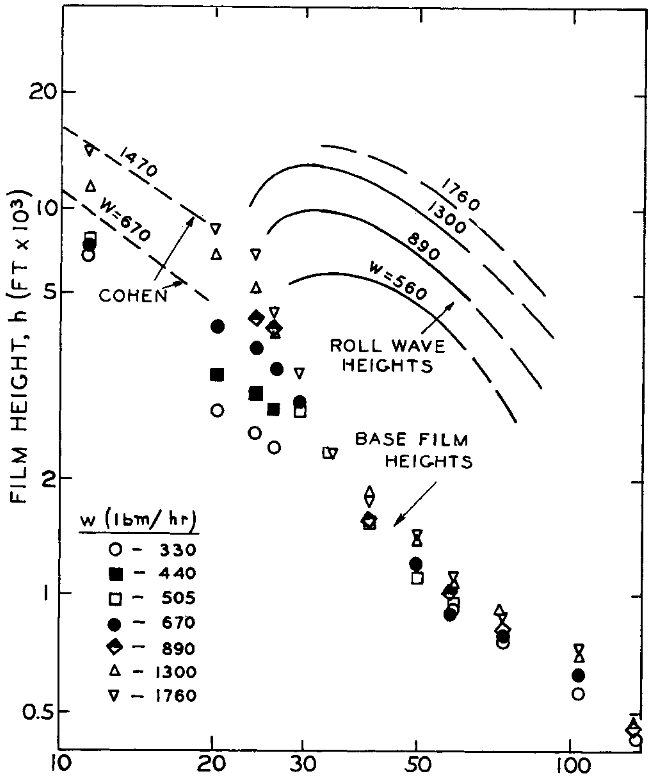

Fig. 2. Superficial air velocity U_S (ft./sec.)

In Figure 3, the dimensionless base film height h_o^+ is plotted against the superficial gas velocity. It is to be noted that a behavior at large gas velocities is attained whereby h_o^+ becomes relatively insensitive to changes in the gas velocity and appears to be approaching a limiting value of about 12. If a linear velocity profile is assumed to exist in the film for small values of h^+ then a value $h_o^+ = 12$ corresponds to a water flow rate of about 155 lb./hr. ft. As reported (*18*) this is equal to critical liquid flow rate needed for the generation of roll waves in this system. It also corresponds to the critical condition reported by Taylor (*16*) for an upward flow of air and water in a vertical pipe and by Charvonia (*2*) for downward flow. It is therefore concluded that at a fixed gas velocity the base film maintains a flow condition for different liquid flow rates corresponding to that which exists at the transition to roll waves.

From the measured shear stresses at the wall friction factors $f = 2\ (u^*/U)^2$ have been calculated and are plotted in Figure 4. The friction factors are somewhat larger than for flow past a smooth wall. This is to be expected since the base film is roughened by small ripples. Charvonia reported pressure drop measurements for air-water flow down a vertical pipe at small enough liquid flow rates that roll waves did not exist on the liquid film. Some of his results are also shown in Figure 4. These are in reasonable agreement with our own results.

RELATED STUDIES

It would be of considerable interest to see how generally

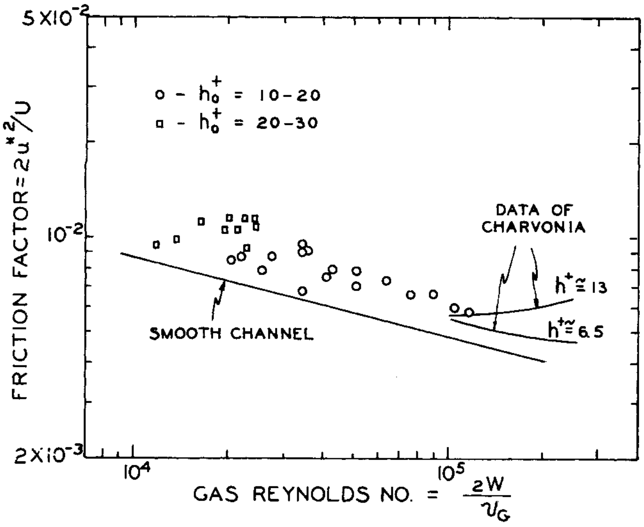

Fig. 4. Friction factors for flow over the base film.

applicable are the numerical results of this research. For this purpose data are needed on other fluids than water and for different orientations of the channel. Some help along these lines is obtained by examining the results on roll wave transition on thin liquid films. Measurements of the critical liquid flow rate for different liquids has indicated that it is relatively insensitive to viscosity (*12, 15, 18*). This would seem to indicate that the limiting value of $h_o{}^+$ will vary with different liquids and that the best way to present this limiting behavior is in terms of the mass flow of liquid per unit length, which has been found to be approximately equal to 155 lb./hr. ft. for water.

There is some similarity in the results of this research and previous studies of air-water flow down a vertical pipe by Chien and Ibele (*3*) and by Charvonia (*2*). Both of these investigators measured fluctuations in the film height. They defined a continuous film, called the *liquid sublayer*, over which waves are moving. They found the thickness of the liquid sublayer to depend only on the gas flow rate and to be somewhat insensitive to changes in the liquid flow rate. The work of Charvonia dealt with annular flows which were not atomizing so that roll waves were not present on the liquid film. The frequencies of the waves that he examined were much larger than the roll waves. The wave characteristics that he measured probably correspond to those which exist on the base film. The liquid sublayer thickness of about 0.002 in. could be interpreted as the troughs of the waves existing on the base film. The base film reported in this paper corresponds to the mean film heights measured by Charvonia. His measured mean film heights at the transition to an atomizing flow compare with our measurements in Figure 2.

The measurements of Chien and Ibele were made mainly in the region of atomizing flow. Their liquid sublayer heights agree quite well with those obtained by Charvonia. However, their average film heights and wave heights are much larger. This would be expected since roll waves were probably present and the average reported by these authors includes both the base film and roll waves.

CONCLUDING REMARKS

The observation that for a given gas rate the base film tends to maintain a flow rate close to that existing in the film at the transition to roll waves has probably been suggested directly or indirectly by a number of researchers. For example, Chung and Murgatroyd (*4*) have implied this in their analysis of the roll wave regime. The present paper gives some experimental support for such a simplifying assumption and is somewhat encouraging with respect to the possibility of giving a more detailed account of flow in the liquid film. The success of future research on the model depends on our ability to take account of the flow in the roll waves.

For a given liquid and gas flow rate one would like to be able to predict the shape, the velocity, the volume of liquid in the roll waves, and the variation of the shear stress and pressure of the gas flowing over the top of the roll wave. Such an analysis has been attempted (*18*) by ignoring the detailed wave structure on the surface of the roll wave and by approximating it as possessing a gradually sloping back and steep front. The flow field in the back was described by a pseudosteady state analysis similar to that presented in equation (7) of reference 9. The properties of the solution of the resulting differential equations are difficult to determine because of the large number of unknown parameters; for example, the relationship of the local wall shear stress to the local volumetric flow rate in the liquid, the shape of the liquid velocity profile, the pressure and shear stress at the interface between the liquid and the gas. At present it is not clear what are the defining differential equations for a roll wave.

ACKNOWLEDGMENT

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NOTATION

C = capacitance
f = friction factor = $2\ (u^*/U)^2$
h_o = height of base film
h^+ = dimensionless height = hu^*/ν
I = electrical current
$N_{Re}{}^L$ = Reynolds number of liquid = w/ν
$N_{Re}{}^G$ = Reynolds number of gas $= 2W/\nu_G$
R_f = resistance of heat film
R_{std} = standard resistance
R_{var} = variable resistance
R_x = resistance of the unknown
T_f = temperature of heated element
T_L = temperature of liquid
u^* = friction velocity
U = bulk averaged velocity of the gas
U_s = superficial gas velocity, based on the height of the empty channel
W = weight rate of flow of gas per unit width
w = weight rate of flow of liquid per unit width
ν_G = kinematic viscosity of the gas
ν = kinematic viscosity of the liquid
τ_W = shear stress at the wall

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Temperature—Separation Factor Relationships in Gaseous Diffusion

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A theoretical equation was developed to predict the enrichment produced by a single stage porous Vycor diffusion cell as a function of temperature. The separations of five binary gas mixtures were measured experimentally over a temperature range of 80 to 600°K. The agreement between theoretical predictions and experimental results was good over most of the temperature range. The largest deviations occurred when the operating temperature approached the critical temperatures of the gases. Mixtures of oxygen-carbon dioxide and helium-nitrogen were observed to become nonseparative at predicted temperatures. Overall, both separation and separative capacity were observed to increase with a decrease in temperature.

Enrichment calculations for gaseous diffusion systems have been developed for a number of barriers and gas mixtures (*2, 8, 10, 12, 23*); but no attempts were made by these investigators to establish a relationship between temperature and enrichment. Recent publications by Barrer (*1*) and Hwang (*11*) show, however, that pure gas diffusion is temperature dependent, particularly so in the region of the gas critical temperature. Surmising that the diffusion of gas mixtures might also be similarly affected, Hwang used his pure gas data to predict separation factors for binary mixtures. He did not attempt to verify these predictions experimentally. Therefore, based on Hwang's pure gas flow theory, this paper derives an expression relating temperature and enrichment for porous Vycor diffusion cells. Results from numerous experimental runs using five binary gas mixtures at temperatures from 80 to 600°K. are recorded and compared with the derived equation.

THEORETICAL DEVELOPMENT

Equations for calculating the enrichment achieved with single stage permeability cells were originally published for binary mixtures by Weller and Steiner (*24, 25*). Although these well known formulas were derived for semipermeable polymer barriers, they have been shown to be applicable to other barriers as well. Kammermeyer and Brubaker (*7*) adapted them to microporous media and extended the case I, perfect mixing model, to multicomponent systems. In a later publication Naylor and Backer (*17*) presented a much simpler development for the case II, laminar flow model, when large separation factors are encountered. Benedict and Pigford (*3*) accomplished the same goal for the close separations encountered in isotope recovery. A recent extensive study by Breuer (*6*) further suggests that self-mixing within the gas streams also affects the separation, giving an illusion of perfect mixing.

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For this work, two basic assumptions were made; (a) that different membranes made of the same porous Vycor material behave similarly, and (b) that the Weller-Steiner equations for perfect mixing provide a justifiable starting point for the theoretical development and data correlation. Incorporating these assumptions, an equation representative of the W.-S. case I conditions can be generated which is consistent with the experimental design. That is, the molar ratio of the two components in the diffused stream will be:

$$\frac{y}{1-y} = \alpha^* \frac{(x-Py)}{(1-x)-P(1-y)} \tag{1}$$

For a single stage separation, the actual enrichment achieved is indicated by the stage separation factor α, traditionally defined as:

$$\alpha = \frac{y}{1-y} \cdot \frac{1-x}{x} \tag{2}$$

Combining Equations (1) and (2), it is possible to theoretically predict a stage separation factor, α_T:

$$\alpha_T = \frac{y}{1-y} \cdot \frac{1-x}{x} = \alpha^* \frac{(x-Py)}{(1-x)-P(1-p)} \cdot \frac{1-x}{x} \tag{3}$$

It is important to note that α_T of Equation (3) is only a measure of the performance of a single stage operation and not a property of the system. Thus α_T varies with the operating conditions of the cell. For example, when $P = 1.0$, no driving force exists to produce a separation and $\alpha_T = 1.0$. Correspondingly if P approaches zero, that is, diffusion into vacuum, then α_T becomes:

$$\alpha_T = \frac{y}{1-y} \cdot \frac{1-x}{x} = \alpha^* \tag{4}$$

which further illustrates that α_T approaches α^* as a maximum limit.

If Equation (3) is used to predict the magnitude of